

Real Numbers Definition Examples Properties Symbol Chart

1. Euclid's Division lemma:- Given Positive integers a and b there exist unique integers q and r satisfying

$a = bq + r$, where $0 \leq r < b$, where a , b , q and r are respectively called as dividend, divisor, quotient and remainder.

2. Euclid's division Algorithm:- To obtain the HCF of two positive integers say c and d , with $c > 0$, follow the steps below:

Step I: Apply Euclid's division lemma, to c and d , so we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$

Step II: If $r = 0$, d is the HCF of c and d . If $r \neq 0$, apply the division lemma to d and r .

Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HC

3. The Fundamental theorem of Arithmetic:-

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex.: $24 = 2 \times 2 \times 2 \times 3 = 3 \times 2 \times 2 \times 2$

Theorem: LET x be a rational number whose decimal expansion terminates. Then x can be expressed in the form

Of $\frac{p}{q}$ where p and q are co-prime and the prime factorisation of q is the form of $2^n \cdot 5^m$,

where n, m are non negative integers.

Ex. $\frac{7}{10} = \frac{7}{2 \times 5} = 0.7$

1. If the H C F of 657 and 963 is expressible in the form of $657x + 963y - 15$ find x . Definition (Ans: $x=22$)

Ans: Using Euclid's Division Lemma

$$a = bq + r, 0 \leq r < b$$

$$963 = 657 \times 1 + 306$$

$$657 = 306 \times 2 + 45$$

$$306 = 45 \times 6 + 36$$

$$45 = 36 \times 1 + 9$$

$$36 = 9 \times 4 + 0$$

$$\therefore \text{HCF}(657, 963) = 9$$

$$\text{now } 9 = 657x + 963y \quad (-15)$$

$$657x = 9 + 963 \times 15$$

$$= 9 + 14445$$

$$657x = 14454$$

$$x = 14454 / 657$$

$$x = 22$$

2. Express the GCD of 48 and 18 as a linear combination. (Ans: Not unique)

$$A=bq+r, \text{ where } 0 \leq r < b$$

$$48=18x+12$$

$$18=12x+6$$

$$12=6x+0$$

$$\therefore \text{HCF}(18,48) = 6$$

$$\text{now } 6=18-12x$$

$$6=18-(48-18x)$$

$$6=18-48x+18x$$

$$6=18x-48x$$

$$6=18x+48(-1)$$

$$\text{i.e. } 6=18x+48y$$

$$\therefore \boxed{x=3, y=-1}$$

$$6=18 \times 3 + 48 \times (-1)$$

$$=18 \times 3 + 48 \times (-1) + 18 \times 48 - 18 \times 48$$

$$=18(3+48) + 48(-1-18)$$

$$=18 \times 51 + 48 \times (-19)$$

$$6=18x+48y$$

$$\therefore \boxed{x=51, y=-19}$$